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MATHEMATICAL DYNAMIC MODEL FOR LONG-TERM DISTRIBUTION SYSTEM PLANNING

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Abstract - The presented model gives optimal values of all distribution system design parameters for a consumption area without existing network, in which uniform load density increases with time. The model belongs to a class of nonlinear mixed integer programming tasks, subject to nonlinear constraints. Transformation of dynamic problem into nondynamic form is performed using a defined planning strategy. Objective function of the model is the present value of system investment, maintenance and energy loss costs within a planning period. The constraints, among others (thermal, integer), are the permitted voltage drops in lines. The solution algorithm is described and results of model application for a rural area are given.

INTRODUCTION

Numerous papers |1| deal with distribution system planning models which can be used successfully, mainly for static tasks. Only a few relate to complex models under dynamic load condition. Some are complicated to apply |2|, or do not consider all system variables |3|. The recent paper of Gönen and Foote |4| is promising, but the presented model is limited to smaller problems, due to numerous variables and constraints. Besides, the model includes neither voltage levels optimization nor voltage drop constraints.

The static planning task is to design an optimal distribution system for a particular load density, expected after a pre-specified time period. It is assumed that the system developes in one step. The presented model relates to a system which develops over a long planning period H (over 15 years), depending on the load density forecast p(t), as a function of time, and according to a predefined development strategy. The system develops through several generations of system elements. The time period in which the first generation satisfies the system demand is a variable, whose optimal value is to be found. This period heavily depends on the load density rate of growth and other system parameters and may significantly differ from arbitrarily chosen values in the static models. Therefore, the static models cannot give an optimal system plan, except by chance.

In the presented model uniform load density and constant power factor are assumed in a given load area for period H. Also, the load area is assumed to be without an existing network. These assumptions significantly simplified the development of the model which can be very useful in solving many real planning tasks, particularly in system development of new urban areas. The model gives optimal values of all system design parameters: voltage levels, substation sizes and their service areas, number of feeders, conductor cross-sections and lengths at each level and periods in which system elements should be put into service. The model also gives the nominal and present value of all investment, maintenance and loss costs in any period within H.

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Model structure

Objective function is the density of present value costs of investment, maintenance and energy losses in period H. (For easier comparisson of solutions for different load areas, costs density is used instead of total costs). Each network level j has a corresponding level in an objective function. A level consists of substations (SSTs) and their secondary lines. The lowest is the zero level (j=0) to which loads are connected.

The number of levels J (j=0,1,...,J) is not a variable in the model, and its optimal value is determined by solving models with different numbers of levels and comparing the solutions.

The objective function contains seven dynamic cost functions for each level: two for SST and feeder investment costs, two for their maintenance costs and three for losses: in transformer core and copper and in the feeders. The functions are analytic or tabular. Tables are used when costs cannot be represented precisely by analytic functions.

Beside constraints of nonnegativity and discreteness of variables and constraints concerning the overloading of transformers and feeders, the model also includes voltage drop constraints.

Mathematically, the model is nondynamic, as system variables are not functions of time, although some variables have a dimension of time (the year in which network elements are made operational). Transformation of a dynamic problem into nondynamic model of mathematical programming is performed using the defined system development strategy. The model belongs to the class of mixed integer nonlinear programming tasks, with nonlinear constraints. The main steps of the solution algorithm are described.

A major part of the paper deals with derivation of dynamic costs functions. For clarity, static functions are given first and then the dynamic functions. An example of model application is given at the end of the paper.

STATIC COST FUNCTIONS

Feeder construction costs, I_{fj}

$$I_{fj} = C_j L_j \tag{1}$$

$$C_j = C_{jo} + C_{j1} S_j + C_{j2} V_j \tag{2}$$

where C_{jo}, C_{j1} and C_{j2} are empirical constants and L_j, S_j and V_j represent the length, cross section and voltage of j level feeders, respectively.

The feeder length L_j is calculated for area A_j, supplied by V_{j+1}/V_j SST, and is dependent on the scheme applied and the layout of feeded points. For schemes in Fig. 1, which can be applied to medium voltage levels in city distribution systems, a good approximation of feeder lengths can be obtained by

$$L_j = A_{j-1}^{1/2} N_{fj} \tag{3}$$

where $N_{fj} = h_j(n_j + n_j^{1/2} q_j/4)$

$n_j = A_j/A_{j-1}$

q_j = number of secondary feeders per SST

h_j = empirical constant ($\simeq$ 1.5)

Other formulae have been derived for rural areas and for other levels and schemes.

Substation construction cost, I_{tj}

$$I_{tj} = a_{jo} + r_j(a_{j1}V_{j+1} + a_{j2}P_{tj}) + q_j I_{bj} \tag{4}$$

$$I_{bj} = b_{jo} + b_{j1}V_j + b_{j2}P_{tj}$$

where a_{jk} and b_{jk}, k = 0,1,2 are empirical constants

r_j = number of transformers in a j level SST

I_{bj} = feeder bay construction costs in the same SST

P_{tj} = rated load (kVA) of j level transformer.

Linear functions (4) apply to a limited range of changes in V_j and P_{tj} and must be corrected if the optimal solution is outside those limits. The latter function does not correctly represent the real I_{bj} costs, hence they are usually given in a two dimensional table.

Transformer losses

Power losses in a transformer, as in |5|, depend on $P_t^{3/4}$, which can be verified from manufacturers data. Hence, the power loss in transformer core is

$$P_c = K_c P_t^{3/4}$$

and in copper at rated load

$$P_{cu} = K_{cu} P_t^{3/4}$$

where K_c = core loss coefficient $W/VA^{3/4}$ ($\simeq$ 0.048)

K_{cu} = copper loss coefficient $W/VA^{3/4}$ ($\simeq$ 0.310)

The present value of core loss cost in a period t_j for a j level SST, reduced to a unit of area A_j is

$$E_{cj1} = \frac{E_{cj}}{A_j} \sum_{t=1}^{t_j} a^t \tag{5}$$

where $E_{cj} = C\, K_c\, 8760\, r_j\, P_{tj}^{3/4}$

C = price of electricity

a = 1/(1+d)

d = discount rate

Henceforth, the present value of costs reduced to a unit area will be referred to as DD costs - Density of Discounted costs.

If t_j is the year in which SST is nominally loaded then DD costs of transformer copper losses in period t_j are

$$E_{cuj1} = \frac{E_{cuj}}{A_j} \frac{1}{p^2(t_j)} \sum_{t=1}^{t_j} p^2(t) a^t \tag{6}$$

where $E_{cuj} = C\, K_{cu} T_{tj}\, r_j\, P_{tj}^{3/4}$

T_{tj} = equivalent hours for j level transformers

p(t) = load density forecast VA/m^2

Feeder losses, E_{fj}

The general formula for DD loss costs in period t_j in area A_j is

$$E_{fj} = \frac{C_{\ell j}}{A_j} M2_j \sum_{j=1}^{t_j} p^2(t) a^t \tag{7}$$

$$\text{where } C_{\ell j} = h_j C\, \rho\, T_N\, e_{fj}/(S_j V_j^2 K_N^2) \tag{8}$$

ρ = electrical resistivity

T_N = equivalent hours for a load duration curve of a sufficiently large area

K_N = contribution factor, representing the participation of maximum load P_N, on the above duration curve, in the total system peak load ($P_N \simeq$ 100 MVA)

e_{fj} = loss increase factor, as transmitted power at levels j>0 is greater than p(t) ($e_{fj} \geq 1$)

$M2_j = \Sigma \ell_i A_i^2$ - second loss moment

A_i = area from which the load P_i arises, flowing through feeder branch of length ℓ_i

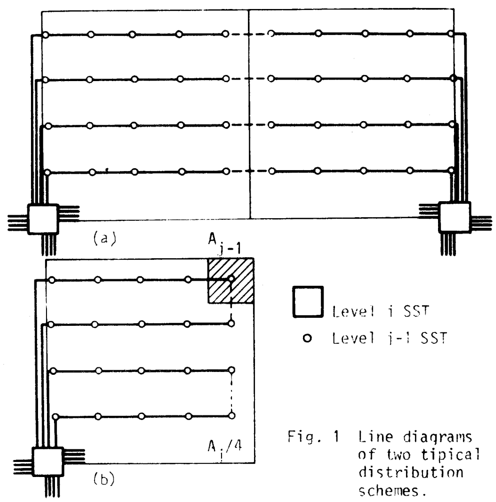


Fig. 1 Line diagrams of two tipical distribution schemes.

The second loss moment $M2_j$ for distribution schemes as in Fig. 1 is

$$M2_j = A_{j-1}^{5/2} n_j^2 \left\{ \frac{n_j}{3q_j^2} + \frac{1}{2q_j} \left[1 + n_j^{1/2}\left(\frac{1}{2} - \frac{2}{q_j}\right) \right] + \frac{1}{n_j} \right\} \tag{9}$$

where $n_j = A_j/A_{j-1}$

Maintenance costs, M_{tj} and M_{fj}

Yearly maintenance costs for SSTs and feeders are given as fixed percentages m_{tj} and m_{fj} of corresponding investment costs. Their calculation for a period t_j is as for transformer core losses. DD maintenance costs of a SST are

$$M_{tj} = m_{tj} \frac{I_{tj}}{A_j} \sum_{t=1}^{t_j} a^t \tag{10}$$

and DD feeder maintenance costs

$$M_{fj} = m_{fj} \frac{I_{fj}}{A_j} \sum_{t=1}^{t_j} a^t \tag{11}$$

where I_{tj} and I_{fj} result from (4) and (1).

DISTRIBUTION SYSTEM DEVELOPMENT STRATEGY

The strategy is defined here as a set of rules (algorithms) for investment decision making dependent on the state of the system. The planner's task is to find an optimal strategy according to optimality criteria. The rules suggested here are influenced both by technical constraints and by interest of utilities to achieve economy of scale in design, construction and maintenance of the system. These rules are:

1. All voltage levels are constant in the planning period (horizon) H.

2. All SSTs and all line cross sections in the area with the same load density and voltage level are of the same size in period H.

3. Construction planning is in phases. A phase is a period in which load density and the number of SSTs doubles. This does not apply to the first phase whose duration is a variable in the model. Phase durations at different system levels can differ. One or more generations of system elements are constructed in each phase, except in the first. All elements made operational at the same time (ideally) belong to a generation. The first phase has only one generation, which takes on the entire load of the area at the beginning of the planning period. The second and all other phases have the same number of generations except, may be, the last.

4. At the end of each phase, corresponding network elements are fully loaded (ideally).

5. SSTs of a new phase are built to reduce the existing SSTs load. To minimize investment costs, a minimum number of network elements should be constructed in each generation, providing the existing ones do not become overloaded. So, in an optimal development plan, there should be a maximum number of generations phisically possible, in all phases after the first.

6. Elements constructed within the same phase do not unload each other, but the elements built in earlier phases. This is obvious if rule 5. is obeyed, as the SST areas are not adjacent.

7. According to rules 2. and 4, the area supplied by a new phase SST is half of the SST area from previous phases.

With this strategy dynamic cost functions can be defined, in which system variables are independent of time. This enables transformation of the optimization problem into a nondynamic form, which is easier to solve.

DYNAMIC COST FUNCTIONS

SST investment costs in period H, I_{tjH}

If the life of a SST is D years, then DD depreciation costs for SSTs of level j, in period H, are

$$I_{tj1} = \frac{I_{tj}}{A_j}(a^t - R(t)) \tag{12}$$

where t = the year in which SST was installed
I_{tj} = obtained from (4)
$R(t) = (D-H+t)a^H/D$ - residual value of SST installed in year t, at the end of period H, per unit of I_{tj}.

After the first generation of SSTs, made operational at t=0, became saturated in year t_j, new SSTs need to be available, one for each area of size G A_j, where G is the number of adjacent SSTs whose load can be reduced by one new SST. After building G generations, the total number of SSTs doubles, so completing the second phase. In the third phase everything is repeated except that one SST of each new generation is built in area G $A_j/2$. In general, the area size in which one SST of a new generation in phase e is built is

$$A_{je} = G\,A_j/2^{e-2} \tag{13}$$

Let us denote the year in which the SST from phase e and generation g is made operational with t_{eg}. The total DD costs of all SSTs in period H are

$$I_{tjH} = \frac{I_{tj}}{A_j}\Big[1 - R(o) + \frac{1}{G}(a^{t_j} - R(t_j)) + .. + \frac{2^{e-2}}{G}(a^{t_{eg}} - R(t_{eg})) + .. + \frac{2^{E-2}}{G}(a^{t_{Eg}} - R(t_{Eg}))\Big] \tag{14}$$

or

$$I_{tjH} = I_{tj}[1 - R(o) + S_t]/A_j \tag{15}$$

where

$$S_t = \frac{1}{G}\sum_{e=2}^{E} 2^{e-2} \sum_{g=1}^{G} (a^{t_{eg}} - R(t_{eg})) \tag{16}$$

E = total number of phases in period H

The variable t_{eg} is not independent because it can be calculated according to previously defined strategy, if the duration of the first phase t_j and the load density forecast p(t) are given. Building each new generation of SSTs in the second phase, the area supplied by the first phase SST is reduced by $A_j/(2G)$, i.e., for g-1 generations by $(g-1)A_j/(2G)$. When the g-th generation is put into service, the relation

$$(1 - \frac{g-1}{2G})A_j p(t_{2g}) = A_j p(t_j)$$

or

$$p(t_{2g}) = \frac{2G}{2G+1-g}\,p(t_j) \text{ applies.}$$

In the third phase, as already stated, the area which is being reduced is half of A_j, so that

$$p(t_{3g}) = \frac{2G}{2G+1-g}\,2\,p(t_j)$$

and in any phase e will be

$$p(t_{eg}) = \frac{2G\,2^{e-2}}{2G+1-g}\,p(t_j) \tag{17}$$

Using (17) t_{eg} can be calculated for each value of t_j, e and g, and I_{tjH} by (15) as a function of t_j. In the optimization procedure, optimal values of t_j are determined for each j. Values for S_t are given in Table I for $p(t) = 23.4 + 1.8\,t + 0.072t^2$ VA/m^2, H = 30 yrs, d = 10% and G = 4.

TABLE I. Dynamic terms of cost functions

t_j yrs	$p(t_j)$ VA/m^2	S_t	$\underline{S}_t$	S_{cu}	$\bar{S}_{cu}$
1	2	3	4	5	6
5	34.200	0.7146	0.5318	8.6113	9.9176
6	36.792	0.6160	0.4540	7.5056	8.6626
7	39.528	0.5324	0.3869	6.5261	7.5593
8	42.408	0.4605	0.3291	5.6898	6.5898
9	45.432	0.3967	0.2793	4.9606	5.7382
10	48.600	0.3387	0.2365	4.3093	4.9901
11	51.912	0.2899	0.1997	3.7527	4.3331
12	55.368	0.2457	0.1682	3.2554	3.7560
13	58.968	0.2072	0.1412	2.8171	3.2490
14	62.712	0.1751	0.1181	2.4388	2.8036
15	66.600	0.1450	0.0984	2.0907	2.4123

It is interesting to find the lower limit of cost I_{tjH}, ie, $\underline{S}_t$. It would be reached if there was no surplus installed capacity at any time after the first phase. This assumes a hypothetical strategy for which it is either possible to build new SSTs in arbitrarily small increments in a specified area or that a new SST can take on the load increase in an arbitrarily large load area, in a very short time. In both cases the system can be considered to have one more phase after the first with unlimited number of generations. DD costs of the new SST increment, for the interval t_g, t_{g+1}, in area

A_j are

$$\frac{I_{tj}}{p(t_j)}\,[p(t_{g+1}) - p(t_g)]/A_j$$

and using (14)

$$I_{tjH} = \frac{I_{tj}}{A_j}(1 - R(o)) + \frac{I_{tj}}{A_j p(t_j)}\sum_{g=1}^{G}(a^{t_g} - R(t_g))(p(t_{g+1}) - p(t_g)) \qquad (18)$$

If $G \to \infty$, then $p(t_{g+1}) - p(t_g) \to p'(t)\,dt$, and (18) becomes

$$I_{tjH} = \frac{I_{tj}}{A_j}\left[1 - R(o) + \frac{1}{p(t_j)}\int_{t_j}^{H}(a^t - R(t))p'(t)dt\right] \qquad (19)$$

If follows from (19) that

$$\underline{S}_t = \frac{1}{p(t_j)}\int_{t_j}^{H}(a^t - R(t))p'(t)dt \qquad (20)$$

Column 4 of Table I shows the calculated values of $\underline{S}_t(t_j)$.

According to (15) the dynamic cost function is obtained from the static, calculating an additional function S_t, which is dependent only on t_j - the saturation period of the first phase SSTs. In the optimization procedure it is useful to calculate S_t in advance for $t_j = 0,1,2,\ldots,H$, which significantly speeds up the procedure.

Transformer core losses, E_{cjH}

According to (5), energy losses in transformer core, for first phase SSTs in period H, are

$$E_{cj1} = \frac{E_{cj}}{d\,A_j}(1 - a^H)$$

Analogous to (15), total costs for all phases in period H will be:

$$E_{cjH} = \frac{E_{cj}}{d\,A_j}(1 - a^H + S_c) \qquad (21)$$

where

$$S_c = \frac{1}{G}\sum_{e=2}^{E}2^{e-2}\sum_{g=1}^{G}(a^{t_{eg}} - a^H) \qquad (22)$$

and where t_{eg} is calculated from (17).

Equation (22) is similar to (16) because $S_c = \lim_{D\to\infty} S_t$. The same applies to $\underline{S}_c$.

SST maintenance costs, M_{tjH}

Based on (10) and (21), total DD maintenance costs for level j SSTs are

$$M_{tjH} = \frac{I_{tj}m_{tj}}{d\,A_j}(1 - a^H + S_c) \qquad (23)$$

Transformer copper losses, E_{cujH}

Copper losses must be calculated separately for each interval t_{eg}, $t_{e,g+1}$ as the loading of SSTs is permanently changing. Also, losses must be separately determined for SSTs built before and during phase e, as they are differently loaded. Analysing the loading of these SSTs at each interval and considering their increasing number in area A_j, the equation for calculating DD transformer copper loss costs in period H is

$$E_{cujH} = \frac{E_{cuj}}{A_j}\left[\frac{1}{p^2(t_j)}\sum_{t=1}^{t_j}p^2(t)a^t + S_{cu}\right] \qquad (24)$$

where

$$S_{cu} = \frac{1}{p^2(t_j)}\sum_{e=2}^{E}2^{-e}\sum_{g=1}^{G}\left[(2-\tfrac{g}{G})^2+\tfrac{g}{G}\right]\int_{t_{eg}}^{t_{e,g+1}}p^2(t)a^t dt \quad (25)$$

For a hypothetical planning strategy, described in deriving (20), transformer copper losses would reach the upper limit, as all SSTs would be fully loaded. It is easy to show that this limit for S_{cu} is

$$\bar{S}_{cu} = \frac{1}{p(t_j)}\int_{t_j}^{H}p(t)a^t dt \qquad (26)$$

The values for S_{cu} and $\bar{S}_{cu}$ are given in Table I. Their ratio is relatively stable for different values of t_j, and for already given p(t) amounts to $S_{cu}/\bar{S}_{cu} \approx 0.866$.

Feeder investment costs, I_{fjH}

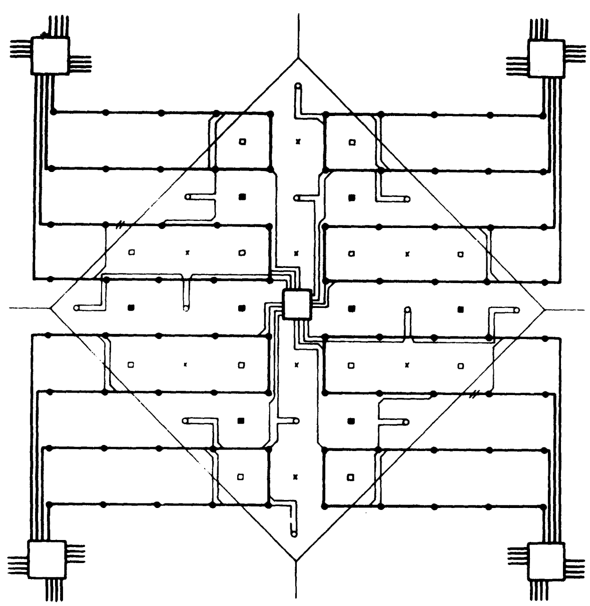
Determination of a dynamic function for feeder investment costs is more difficult than for SSTs, as each new SST generation causes network reconstruction. This particularly applies to a medium voltage level lines unlike the low or high voltage lines, whose reconstruction is not always necessary. Hence, one distribution scheme suitable for medium voltage levels will be analysed here.

If the life of feeders is D_f, then DD depreciation costs in period H for feeders installed at $t = t_{eg}$ are

$$I_{fjeg} = L_{je}C_j\,(a^{t_{eg}} - R_f(t_{eg}))/A_{je} \qquad (27)$$

where L_{je} = the length of feeders constructed in any generation of phase e and is independent of g
A_{je} = given by (13)
$R_f(t_{eg})$ = $(D_f - H + t)\,a^H/D_f$, the residual p.u. value of feeders at the end of period H, installed in year t_{eg}

The length L_{je} must be determined for each distribution scheme. Fig. 2 illustrates the network reconstruction for the scheme in Fig. 1-(b). New lines are indicated by thin lines. Apart from lines required to connect existing feeders to a new SST, new lines are required to connect new SSTs of a lower level (j-1) to level j feeders. Their number n_j in area A_{je} always remains the same (is independent of e and g). Only the distance between j-1 level SSTs changes, dependent on the area they supply. This area in each new phase is



● First phase SSTs　　○ SSTs of first generation in second phase
-//- Unused lines

Fig. 2 Example of reconstructed scheme from Fig. 1-(b) half the area of the previous phase. Therefore, a formula, similar to (3), can be used for L_{je}

$$L_{je} = (\frac{A_{j-1}}{2^{e-1}})^{1/2} N_{fj1} \quad (28)$$

where $N_{fj1} = h_j[n_j + n_j^{1/2}(\frac{q_j}{4} + 2)]$

The difference in relation to (3) is that $q_j/4$ in (3) is replaced by $q_j/4+2$ in (28), which is in accordance with empirical results. As (28) holds for all generations of the second phase, and similar conditions in network reconstruction exist in other phases, the total DD investment cost of level j feeders are

$$I_{fjH} = \frac{A_{j-1}^{1/2}}{A_j} [C_j N_{fj}(1-R_f(o)) + C_{j1}N_{fj1}S_f] \quad (29)$$

where $S_f = \frac{1}{2G} \sum_{e=2}^{E} 2^{(e-1)/2} \sum_{g=1}^{G} (a^{t_{eg}} - R_f(t_{eg})) \quad (30)$

Two different unit prices for line construction are given in (29): C_j, for the first generation and C_{j1} for others. Different prices appear in network development of a new settlement, since the construction costs of the first generation are significantly lower than the costs of latter generations (digging up streets, ensuring traffic flow, limited timescales for work completion etc.).

Calculation of S_f is the same as for S_t, except that D is replaced by D_f and 2^{e-1} by $2^{(e-1)/2}$.

Equation (29) for I_{fjH} is also valid for the scheme in Fig. 1-(a) (tapped tie lines).

As earlier, a formula for $\underline{S}_f$, the lower limit of S_f, can be derived, applying a hypothetical strategy

$$\underline{S}_f = p^{-1/2}(t_j)\int_{t_j}^{H} (a^t - R_f(t))p^{1/2}(t)\, p'(t)dt \quad (31)$$

Feeder maintenance costs, M_{fjH}

This cost function can be directly derrived from corresponding investment costs. Maintenance cost function for the case represented by (28) is

$$M_{fjH} = -\frac{m_{fj}A_{j-1}^{1/2}}{d\, A_j} \left[C_j N_{fj}(1-a^H) + C_{j1}N_{fj1}S_{fm}\right] \quad (32)$$

where

$$S_{fm} = \frac{1}{2G} \sum_{e=2}^{E} 2^{(e-1)/2} \sum_{g=1}^{G} (a^{t_{eg}} - a^H) \quad (33)$$

Feeder losses in period H, E_{fjH}

Equation (7) can be used for calculating only the losses in the first phase. The calculation is significantly more difficult for other phases, as the structure of the network changes.

Loss moment $M2_j$ from (7) is a function of A_j, A_{j-1} and q_j. As e and g change so do A_j and A_{j-1} causing different loading of feeders, so that even q_j cannot be considered constant. Hence, the loss calculation is complex and will not be shown here. Only the final expressions are given.

The loss calculation is greatly simplified if it is assumed that all SSTs of the same level supply equal areas $\bar{A}_{jeg}$ at any time and that all feeders are equally loaded. Applying the planning strategy, this average area for given e and g is

$$\bar{A}_{jeg} = 2^{2-e}\, G\, A_j/(G+g) \quad (34)$$

Assuming that j and j-1 level SSTs are put into service by the same schedule ($t_{jeg} = t_{j-1,eg}$), then

$$\bar{A}_{jeg}/\bar{A}_{j-1,eg} = A_j/A_{j-1}$$

It follows from (9) and (34) that the loss moment $M2_{jeg}$ is

$$M2_{jeg} = G^{5/2}[2^{e-2}(G + g)]^{-5/2}\, M2_j \quad (35)$$

and the total losses are:

$$E_{fjH} = C_{\ell j} \frac{M2_j}{A_j} \left[\sum_{t=1}^{t_j} p^2(t)a^t + S_\ell\right] \quad (36)$$

where

$$S_\ell = \sum_{e=2}^{E} 2^{(2-e)3/2} \sum_{g=1}^{G} (\frac{G}{G+g})^{3/2} \int_{t_{eg}}^{t_{e,g+1}} p^2(t)a^t dt \quad (37)$$

Equation (36) is not precise because of the mentioned assumptions. In order to calculate the losses more accurate, one dynamic function similar to (37) will have to be determined for each of the four possible exponents of n_j in the formula for $M2_j$. For brevity, these functions are not presented here. Comparing the accurate results with those of formula (36), the latter are 16% smaller. Introducing a correction coefficient in (36) we get

$$E_{fjH} = C_{\ell j} \frac{M2_j}{A_j} \left[\sum_{t=1}^{t_j} p^2(t)a^t + 1.16\, S_\ell\right] \quad (38)$$

which is sufficiently precise for most applied distribution schemes. The upper limit of S_ℓ for a hypothetical strategy would be

$$\bar{S}_\ell = p^{3/2}(t_j) \int_{t_j}^{H} p^{1/2}(t)a^t dt \quad (39)$$

The ratio $S_\ell(t_j)/\bar{S}_\ell(t_j)$ for a previously given p(t), varies slightly with t_j and is about 0.780, which means that even function (39) can be used for calculating total losses, with a corrective coefficient of 1.16×0.780 = 0.905.

THE MODEL

Objective function

Objective function is the sum of all derived dynamic cost functions for each j

$$F = \sum_{j=o}^{J} (I_{tjH}+M_{tjH}+I_{fjH}+M_{fjH}+E_{cjH}+E_{cuJH}+E_{fjH}) \quad (40)$$

The highest level J usually has subtransmition lines only, and the goal function does not contain corresponding SST costs.

System variables

Independent variables are t_j, V_j, r_j, P_{tj} and S_j. Continual variables are t_j, the others are discrete. Variables V_j can also be continual if an optimal, non-standard voltage value is required. Variables t_j are not required for each system level. Two variables are usually sufficient, t_0 and t_1, where the first refers to the zero level and, eventually, to V_1 level lines, and the latter to the rest of the system. Dependent (or derived) variables are numerous but only A_j and q_j are used in calculating the objective function:

$$A_j = r_j P_{tj} K_{tj}/[p(t_j)\, g_{tj}] \quad (41)$$

$$q_j = \text{Integer of } \left[f_s r_j P_{tj} K_{tj}/[\sqrt{3}V_j I(S_j)K_{fj}]+0.5\right] \quad (42)$$

where K_{tj}, K_{fj} = contribution factors of transformer and feeder peak loads, respectively, to the system peak load in year t_j

g_{tj} = the peak power loss coefficient ≥ 1, indicating system peak power losses in system elements below j level SST

$I(S_j)$ = feeder Amps capacity, whose cross section is S_j
f_s = security factor (f_s may be 2 for schemes on Fig. 1).

Constraints

Beside constraints of nonnegativity and discreteness of variables, and constraints concerning the overloading of transformers and feeders, voltage drop constraints are introduced

$$VD_j \leq VP_j \tag{43}$$

where VD_j and VP_j are the maximal and the permitted voltage drops in j level feeders, respectively.

ALGORITHM DESCRIPTION

The algorithm used is a combination of total search and cyclic coordinate descent method. Total search relates to variables P_{to} and P_{t1}. An optimal solution is obtained for each combination of transformer ratings at levels 0 and 1, which is very suitable for post-optimality analysis. Cyclic coordinate descent method is applied to remaining variables. The steps in the recomended algorithm are:

1. Calculate dynamic functions S_t, S_c, S_{cu}, S_f, S_{fm} and S_ℓ for all values $t = t_{min} + n\,(\delta t)$, $(0 \leq t_{min} \leq t \leq t_{max} \leq H$; $n = 0,1,2,\ldots$; usually $(\delta t) = 1/2$ year).
2. Select the combination of transformer rated loads, P_{to} and P_{t1}.
3. For these values and an arbitrary starting point X in the space of the remaining variables, calculate the goal function F using a routine which also checks the voltage drop constraints, as described below.
4. Vary one coordinate of point X, keeping the others constant while F is decreasing. Repeat this process for each coordinate. Go to 4 as long as F decreases, else go to step 5. Coordinates t_j and V_j (if V_j should not have standardized values) are varied with variable increments Δt_j and ΔV_j, and r_j with a fixed increment $\Delta r_j = 1$. Variables S_j (as well as R_{tj}) are read from tables of their rated values.
5. Check if the decrease of F in step 4, ΔF, is greater than a selected number ε. If it is not, go to step 7.
6. Check if $\Delta t_j > (\delta t)$ or $\Delta V_j > (\delta V_j)$, $(j = 0,1,\ldots,J)$ where (δV_j) is a minimal increment of voltage at level j. If none of these conditions is satisfied go to step 7. Otherwise, decrease the corresponding increments so that their new values are $\Delta t_j - (\delta t)$ and $\Delta V_j/2$. Continue with step 4. Note that t_j and Δt_j must be an integer times greater than (δt) and that Δt_j cannot be less than (δt) (see step 1).
7. Optimisation is completed for selected combination of values P_{to} and P_{t1}. Check whether that is the last combination; if not, go to step 2.

This algorithm does not guarantee a global optimum except when voltages are constant. When voltage optimisation is required, then the solution should be verified by additional runs using different fixed values of V_j.

Calculation of objective function

Parallel to calculating the value of the objective function, voltage drop constraint is verified. For given initial point and j=0, the steps are as follows:

1. Calculate values A_j and q_j using (41) and (42).
2. Calculate the value of maximum voltage drop VD_j and check the constraint (43) ($VD_j \leq VP_j$).
3. If the constraint is satisfied and j=J, the calculation ends in step 5. If $j < J$, return to step 1 for $j = j+1$.
4. If the constraint is not satisfied and $q_j < q_{jmax}$, set $q_j = q_{j+1}$ and return to step 2. If $q_j = q_{jmax}$ and $S_j < S_{jmax}$, the calculation ends in step 6. If $q_j = q_{jmax}$, $S_j = S_{jmax}$ and $j < J$, set $j = j+1$ then return to step 1.
5. Calculate all cost functions for $j = 0,1,\ldots,J$ and their total F (end of calculation).
6. $F = M + VD_j$, where M is a large constant (end of calculation).

Cost functions in (40) are only calculated if the solution is feasible (see steps 3 and 4) or if variables q_j and S_j are at upper limits (even if the solution is not feasible). This is useful in the case where V_j is fixed, when a feasible solution may not exist. However, an optimal solution "nearest" to the voltage drop constraint can be found.

MODEL APPLICATION

This model was applied to five small distribution organizations in Yugoslavia. Their load areas are typical: one or two towns of 30-50 thousand inhabitants with approximately the same number in the surrounding rural area. In planning the system development to year 2000, the basic problem was to decide whether to change the existing 4 level system 110/35/10/0.4 kV to $110/V_1/0.4$ kV and whether V_1 should be 10 or 20 kV. Application of the model for towns has shown that the 3 level system is up to 20% cheaper, depending on load density (5-15 MVA/km^2 in year 2000), and that the optimal value of V_1 is about 15 kV (varies slightly with the load as the voltage drop is below its limit). Considering the existing system, V_1 = 10 kV is obviously an advantage.

For a rural area, with chaotic distribution of populated zones, which occupy 10-12% of the total area and whose load density forecast is 140-420 kVA/km^2 in year 2000, the 3 level system is between 11-14% cheaper, but the optimal value of V_1 is between 24-27 kV. Because of the extremely high voltage drop, the 10 kV voltage is unsatisfactory. It is impossible to reduce the size of a 110/10 kV SST below 2 x 20 MVA, as the smaller transformers of this level are nonstandard. Analysis have shown that merging the 110/20/0.4 kV with the existing system up to year 2000 would be more costly than the mentioned savings, and the investors decided to keep the existing system in rural areas. Table II shows the results of the model for load density forecast

$$p(t) = 15.652 + 1.365\,t + 0.308\,t^2 \ kVA/km^2$$

(t = 0 for year 1982)

Other important input data: d = 6%, C = 2 din/kWh, D = D_f = 50 yrs, H = 30 yrs, G = 4, p.f. = 0.9. The network for 0.4; 10 and 20 kV is on concrete poles, radial and for 35 and 110 kV, tapped tie lines (as in Fig. 1-(a)) on towers, aluminium-steel conductors. Dec. 1982 prices (1 $ USA = 52.438 din), VP_0 = 6%, VP_1 = 5%, populated area 10%.

The optimal solution underlined in Table II was used in designing the distribution system, for the area with given load density forecast. The part of the solution refering to subtransmission level is not presented in the table, as it was formed for 3 level system. The cross section of 110 kV lines is $S_3 = 150\ mm^2$. Each line supplies a pair of 110/35 kV, 2 x 20 MVA substations. Their average service area is $A_2 = 1205\ km^2$, supplied by q_2 = 4 35 kV feeders. Other system parameters are given in Table II.

TABLE II. Optimal solutions for distribution system in a rural area, with load density of 280 kVA/km^2 in year 2000

P_{t1} MVA	P_{to} kVA	V_1 kV	VD_1 %	VD_o %	t_o Yrs	t_1 Yrs	S_o mm^2	S_1 mm^2	S_2 mm^2	q_o	q_1	A_o ha	A_1 km^2	F Din/m^2
System 110/35/10/0.4 kV, r_1 = 2														
2.5	50	10	4.8	2.8	13	14	35	35	95	4	7	22.61	161.4	39.078
2.5	100	10	4.2	5.6	16	15	50	35	95	4	6	41.98	145.6	32.125
2.5	160	10	4.8	5.7	20	16	50	35	150	6	5	53.12	132.0	31.022
4.0	50	10	4.9	2.8	13	15	35	35	150	4	11	22.61	237.3	38.674
4.0	100	10	4.7	5.8	20	17	35	35	120	4	9	29.39	195.7	31.412
4.0	160	10	4.5	5.7	20	17	50	35	120	6	8	53.12	195.7	30.597
8.0	50	10	4.7	2.8	13	24	35	50	150	4	16	22.61	226.1	41.272
8.0	100	10	4.9	5.8	20	19	35	50	120	4	14	29.39	334.2	31.877
8.0	160	10	4.7	5.7	20	18	50	50	120	6	13	53.12	364.7	30.996
System 110/V_1/0.4 kV, all variables varied within limits except r_1 = 2														
20.0	50	20	4.8	3.2	12	15	35	35	150	4	28	25.29	1521.3	36.072
20.0	100	23	4.9	5.5	17	17	50	35	150	4	18	38.19	1254.3	27.825
20.0	160	26	5.0	5.7	21	20	50	35	150	6	13	48.99	965.3	26.777
31.5	50	26	4.9	3.2	12	20	35	35	120	4	30	25.29	1533.9	37.758
31.5	100	25	4.9	5.8	21	21	35	35	120	4	24	27.11	1414.7	28.830
31.5	160	27	4.9	5.7	21	20	50	35	120	6	20	48.99	1533.9	27.336
As above except V_1 = 20 kV														
20.0	50	20	4.8	3.2	12	15	35	35	150	4	28	25.29	1521.3	36.072
20.0	100	20	4.9	5.5	17	17	50	35	150	4	21	38.19	1254.3	27.930
20.0	160	20	4.9	5.7	21	19	50	35	150	6	18	48.99	1049.8	26.975
31.5	50	20	4.9	3.2	12	26	35	50	120	4	30	25.29	982.7	39.399
31.5	100	20	4.9	5.8	21	21	35	50	120	4	26	27.11	1414.7	29.021
31.5	160	20	4.7	5.7	21	21	50	50	120	6	23	48.99	1414.7	27.779

CONCLUSION

The presented model is simple to use as it requires minimum of system information and a relatively short computation time. Computation time for one row of Table II is approximately 30" on an IBM 370/135 computer, if there are voltage drop constraints, and if not, then in less than 10". Detailed system information is required to apply the model solution. Using that information a system planner should determine the exact location and boundaries of the service area for each SST,feeder layouts,installation dates for each system element in each phase, equipment specification and investment cost etc. Although it may be time-consuming, the planner´s task is methodologically simple, since all system design parameters are known.

Due to its complexity and short computation time, the model enables sensitivity analyses of the solution to changes in various system parameters, which may be useful tool to a planner.

Two simplifications are introduced in the model: uniform load density and a load area without an existing network. There are no serious difficulties to create a model for two or more load density trends.Only the number of variables and computation time will increase. Let $p_{ij}(t)$ denote i^{th} load density forecast recognized at system level j in area A_{ij} and let F_{ij} be the corresponding part of objective function. The total objective function for the whole area A is

$$F = \frac{1}{A} \sum_j \sum_i F_{ij} A_{ij} \tag{44}$$

Obviously, different load parameters (power, loss and contribution factors) for each $p_{ij}(t)$ can be introduced in model (44).

A similar modelling approach can be applied for a load area with an existing system, but it cannot be explained in a few lines.

In addition to these basic models, there is a need for their different versions as, eg, the versions relating to load area which is gradually populated, according to planned time scales, or to partially populated area, etc. Combining the basic models with their versions gives more complex models, which can be used to solve various real planning tasks.

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